International Journal of Scientific & Engineering Research Volume 9, Issue 5, May-2018 1480
ISSN 2229-5518

Buckling Analysis of Thin Cylindrical Shell Reinforced with Inclined Stiffeners under Uniform Axial Compression using Direct Variational Principle

[1] Ezeh, J.C., [2] Anyaogu, L,[3]Ibearugbulem, O.M., [4] Ettu, L.O, [5] Okoroafor, S. U.
[1,2,3,4,5]Department of Civil Engineering, Federal University of Technology, Owerri, Imo State, Nigeria

Abstract— This research focused on buckling analysis of thin cylindrical shell under uniform axial compression. The method of solution was carried out by the use of direct variational method. The directional method applied in the analysis was the Ritz method. The Ritz method was used to determine the buckling stress parameter of the shell. Numerical examples were carried out with wavelength ratio, deflection parameters, radius of curvature, internal pressure and thickness of the shell kept constant. The results showed that 10^0 inclined stiffeners have the highest buckling stress value at imperfect ratio of 0.5. While 45^0 inclined stiffeners have the least buckling stress value at imperfect ratio of 0.1.

Index Terms— Thin cylindrical shell, buckling stress, axial compression, the Ritz, imperfect ratio, direct varional method, deflection theory.

1 Introduction

A cylindrical shell is generated by moving a straight line along a curve while maintaining it parallel to its original position A thin cylindrical shell is one that maximum ratio of its thickness, h to the radius of curvature, R is less than or equal to 1/20 [1]; [2].

A structure may have two kinds of failure, namely material failure and form failure. In material failure, the stresses in the structure exceed the specified safe limit, resulting in the formation of cracks which cause failure. In form failure, though the stresses may not exceed the safe value, the structure may not be able to maintain its original form. Here, the structure does not fail physically, but may deform to some other shape due to intolerable external disturbance. Furthermore, form failure depends on the geometry and loading of the structure. It occurs when the conditions of loading are such that compressive stresses get introduced. When magnitude of the load on the structure is such that the equilibrium changes from stable to neutral, the load is called the critical load. This phenomenon of change of equilibrium is called the buckling of the structure [3]; [4].

Buckling is often critical in thin-walled or light weight members like slender columns, plates and cylindrical shells which are subjected to predominantly compressive action. Yet the demand for efficient, light weight structures often dictates the use of thin walled members.

- *Ezeh, J.C. is A Professor in Civil Engineering Department in Federal University of Technology Owerri Imo State, Nigeria. E-mail: engrlewy@gmail.com.*
- *Co-Authors are lecturers in Civil Engineering Department in Federal University of Technology Owerri Imo State, Nigeria. E-mail: solomonokoroafor@ymail.com*

This demand is prevalent in the design of silos, liquid retaining structures, aerospace and hydrospace structures. The collapse of a structure like cylindrical shell structures, precipitated by buckling is often a more serious problem than fracture or yielding. Buckling sometime occurs suddenly without warning causing a catastrophic failure. Fracture or yielding, on the other hand, can also produce failure, but the elasticity of the material permits a redistribution of the stresses often allowing a progressive collapse rather than a sudden complete collapse characteristic of buckling. Once buckling is initiated within the structure, there is little or no chance of recovery unless the load is suddenly reduced [4]. Hence, the design of thin cylindrical shells should be based on buckling criteria [5]. Buckling behaviour of cylindrical shells (in particular, the critical buckling load) is not accurately predicted by linear elastic equations due to initial imperfections of the shell structure under the action of compressive loads.

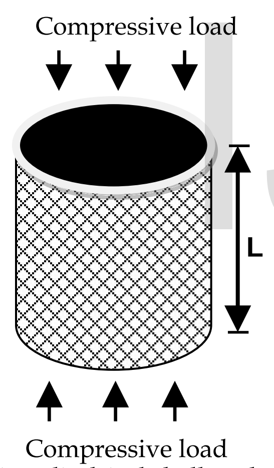
The imperfections include geometrical, structural and loading imperfections. These imperfections affect the load carrying capacity of the shell. The most dominant among these imperfections is geometrical imperfections [6]. The geometrical imperfection is mostly due to deviation in circularity of the shell during its manufacturing. The presence of this imperfection greatly reduces the buckling load predicted for a shell of perfect geometry. Thus, reliable prediction of buckling strength of these shell structures is important, because the buckling failure is catastrophic [7]. The buckling effect on the cylindrical shell structures can be resisted with incorporation of stiffeners in the shell. The circumferential stiffeners are known as ring while longitudinal stiffeners are called stringers [8]; [7].

The main objective of this research is buckling analysis of internally pressurized thin cylindrical shell reinforced with inclined stiffeners under uniform axial compression using direct variational principle. This was achieved by assuming the displacement function of the shell. Its stress function was obtained from the assumed displacement function from the compatibility equation which was carried out by non linear large deflection theory. The expression of the stored energy in the shell as well as work done by the external load was obtained using both the stress and displacement functions. The large deflection terms, effect of imperfection in the strain displacement and the external load were considered in the formulation of total strain energy of the imperfect shell. The resulted total strain energy was minimized using the Ritz method to determine the equation for obtaining the buckling stress values of the shell.

1.1 Direct Variational Methods

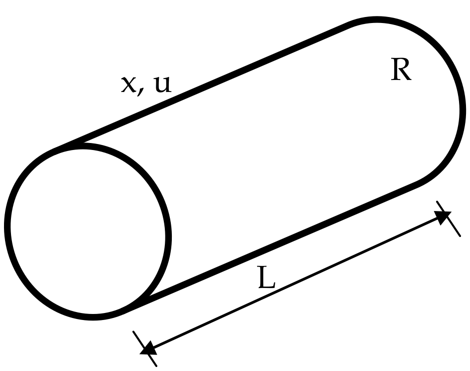
These are methods which (bypassing the derivation of Euler equations) go directly from a variational statement of the problem to the solution. These methods use the following principles: Principle of Conservation of Energy, Principle of Virtual Work and Principle of Minimum Potential Energy for determining numerical fields of unknown functions (i.e. displacement, internal forces and moments) avoiding the differential equations of the plate or shell theory. This method is also called Energy Method. The most widely used ones is the Ritz method [9], [10].

2.0 ENERGY EXPRESSION FOR THE CYLINDRICAL SHELL



(a)

Fig. 1: (a) thin cylindrical shell under axial compression and



(b)

(b) Coordinates and Displacement Components of a point on the Middle- surface of the shell.

Let x and y be the axial and circumferential axis in the median surface of the undeformed cylindrical shell as shown in Fig. 1(a), w is the total radial deflection and w0 represents the initial radial deflection.

From the theory of elasticity, the strain – displacement relations of the cylindrical shell are as expressed in Eqns. (1a), (1b) and (1c) respectively.

$$\epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{dw_0}{\partial x}\right)^2 \quad (1a)$$

$$\epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \frac{w - w_0}{R} \quad (1b)$$

$$\epsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}.\frac{\partial w_0}{\partial y} \quad (1c)$$

The stresses and strains in the middle surface of the shell in the case of plane stress are related to each other by the following equations.

$$\sigma_x = \frac{E}{1-\mu^2}\left(\epsilon_x + \mu\, \epsilon_y\right) \quad (2a)$$

$$\sigma_y = \frac{E}{1-\mu^2}\left(\epsilon_y + \mu\, \epsilon_x\right) \quad (2b)$$

$$\sigma_{xy} = \frac{E}{2(1+\mu)}\epsilon_{xy} \quad (2c)$$

Substituting Eqns. (1a), (1b) and (1c) into their related equations in Eqns. (2a), (2b) and (2c), the followings were obtained;

$$\sigma_x = \frac{E}{1-\mu^2}\left\{\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \mu\left[\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right)\right]\right\} \quad (3a)$$

$$\sigma_y = \frac{E}{1-\mu^2}\left\{\frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - \left(\frac{w-w_0}{R}\right) + \mu\left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]\right\} \quad (3b)$$

$$\sigma_{xy} = \frac{E}{2(1-\mu^2)}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}.\frac{\partial w}{\partial y} - \frac{\partial w_0}{\partial x}.\frac{\partial w_0}{\partial y}\right] \quad (3c)$$

For plane stress state, the non-zero components of stress tensor,

$\sigma_x, \sigma_y, \sigma_{xy}$ satisfied the following equilibrium using Airy stress function F.

$$\sigma_x = \frac{\partial^2 F}{\partial y^2};\ \sigma_y = \frac{\partial^2 F}{\partial x^2};\ \sigma_{xy} = \frac{-\partial^2 F}{\partial x \partial y} \quad (4)$$

Eliminating variables u and v in Eqns. (3) and (4), the relation between stress function F and radial component displacement, w was expressed as follows:

ISSN 2229-5518

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)^2 F$$

$$= E\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}.\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w_0}{\partial x^2}.\frac{\partial^2 w_0}{\partial y^2}-\frac{1}{R}\frac{\partial^2 w}{\partial x^2}+\frac{1}{R}\frac{\partial^2 w_0}{\partial x^2}-\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2\right] \quad (5a)$$

Where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ is called Laplace operator.

$$(\nabla^2)^2\mathrm{F} = \mathrm{E}\left[\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}.\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w_0}{\partial x^2}.\frac{\partial^2 w_0}{\partial y^2}-\frac{1}{R}\frac{\partial^2 w}{\partial x^2}+\frac{1}{R}\frac{\partial^2 w_0}{\partial x^2}-\left(\frac{\partial^2 w_0}{\partial x\partial y}\right)^2\right] \quad (5b)$$

For simplicity, w was assumed to be proportional to w_0.

Thus,

$$Л = \frac{w_0}{w} \qquad (6)$$

Where Л is called imperfection ratio and it is independent of x and y.

With the expression from Eqns (5b) and (6), the compatibility equation was expressed as;

$$\left(\frac{1}{1-Л}\right)\nabla^4\mathrm{F} = \mathrm{E}(1+Л)\left[\left(\frac{\partial^2 w}{\partial x\partial y^2}\right)^2-\frac{\partial^2 w}{\partial x^2}.\frac{\partial^2 w}{\partial y^2}\right]-\frac{E}{R}\frac{\partial^2 w}{\partial x^2} \qquad (7)$$

Where ∇^4 is called Bilharmonic operator.

Equation (7) is the compatibility equation of perfect thin cylindrical shell.

The strain energy of isotropic medium referred to arbitrary orthogonal coordinates was expressed as:

$U = \frac{1}{2}\iiint_{vol}\sigma_{ij}\epsilon_{ij}dvol = \frac{1}{2}\iiint_{vol}[\sigma_x\epsilon_x+\sigma_y\epsilon_y+\sigma_{xy}2\epsilon_{xy}+$ $\sigma xz2\epsilon xz+\sigma yz2\epsilon yzdxdydz$ (8a)

Substituting Eqns. 1(a-c), 2(a-c), 3(a-c) and 4 into Eqn. (8a), we have expressions stated in Eqns. (8) and (9) respectively:

i. The extensional strain energy in the shell was expressed as;

$$U_e = \frac{h}{2E}\int_0^L\int_0^{2\pi R}\left\{\left(\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}\right)^2+2(1+\mu)\left[\left(\frac{\partial^2 F}{\partial x\partial y}\right)^2-\frac{\partial^2 F}{\partial x^2}.\frac{\partial^2 F}{\partial y^2}\right]\right\}dxdy \qquad (8)$$

ii. The work of the external force applied at the ends of the shell

The work of the external force applied at the ends of the shell is the product of the applied compressive force and the change in length of the shell.

$$U_c = P_c * change\ in\ length\ of\ the\ shell \qquad (9a)$$

Thus, $U_c = \sigma_c h\int_0^L\int_0^{2\pi R}\left[\frac{1}{E}\left(\frac{\partial^2 F}{\partial y^2}-\mu\frac{\partial^2 F}{\partial x^2}\right)-\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right]dxdy$ (9)

iii The potential due to the internal pressure, p

$$U_p = \int_0^L\int_0^{2\pi R} p(w-w_0)\,dxdy \qquad (10)$$

The bending strain energy of stiffeners.

Considering Fig. 2, the stiffeners were assumed parallel with the y_1, y_2 coordinates lines and the principal direction of the cylindrical shell coincide with x, y, lines

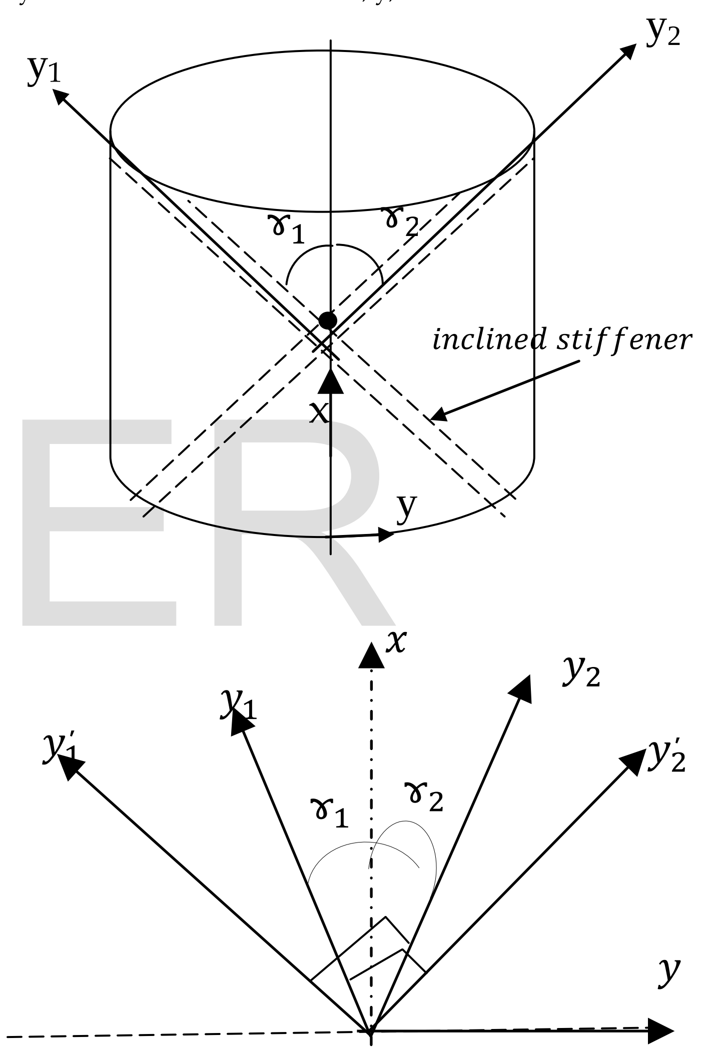


Fig. 2: The coordinate system of the stiffeners of the cylindrical shells and stiffeners

The subscript k was used for k^{th} stiffnener, which is inclined at an angle, γ_1 with generator of the cylinders and is parallel with y_1-line and normal to y_2''– line. Hence, the bending strain energy in the k^{th} stiffener is

$$U_{b,k} = \sum_{k=1}^{N_k}\frac{E_k I_k}{2}\int_0^{L_k}\left(\frac{\partial^2 w}{\partial y_1^2}-\frac{\partial^2 w_0}{\partial y_1^2}\right)^2_{y_2'=0} dy_1 \qquad (11)$$

Where N_k denotes the number of the stiffeners in γ_1– direction.

$E_k I_k$ represents the flexural rigidity of the k^{th} stiffener. The limit L_k is the length of the stiffener in γ_1– direction.
Similarly, the bending strain energy in the j^{th} stiffener which is parallel with y_2– line and nomal to y_1' - line as shown in Fig. 2.

$$U_{b,j} = \sum_{j=1}^{N_j} \frac{E_j I_j}{2} \int_0^{L_j} \left(\frac{\partial^2 w}{\partial y_2^2} - \frac{\partial^2 w_0}{\partial y_2^2} \right)^2_{y_1'=0} dy_2 \qquad (12)$$

The subscript j was used for j^{th} stiffener which is inclined at angle of γ_2 with the generator of the cylinder. Where N_j is the number of the stiffeners in γ_2– direction.

$E_j I_j$ represents the flexural rigidity of the j^{th} stiffeners. The limit L_j is length of the stiffener in γ_2– direction.

v. The torsion strain energy of the k^{th} and j^{th} stiffeners were

$$U_{T,k} = \sum_{k=1}^{N_k} \frac{G_k J_k}{2} \int_0^{L_k} \left[\frac{\partial^2 (w - w_0)}{\partial y_1 \partial y_2'} \right]^2_{y_2'=0} dy_1 \qquad (13)$$

$$U_{T,j} = \sum_{j=1}^{N_j} \frac{G_j J_j}{2} \int_0^{L_j} \left[\frac{\partial^2 (w - w_0)}{\partial y_1 \partial y_2'} \right]^2_{y_1'=0} dy_2 \qquad (14)$$

Where G J represents the torsional rigidity of stiffeners, with subscript j representing stiffeners in γ_2– direction and subscript k is for stiffeners in γ_1- direction. In this analysis, the inclined angles, γ_1 and γ_2 are considered in axial symmetry for inclined stiffeners.

According to Von and Tsien [9], the deflection shape of the cylindrical shell subject to axial compression was assumed as:

$$w = f_1 + f_2 \cos\frac{mx}{R} \cos\frac{ny}{R} + f_3 \cos\frac{2mx}{R} + f_4 \cos\frac{2ny}{R} \qquad (15)$$

Where m and n are the numbers of waves in axial and circumferential directions respectively. The corresponding stress function for cylindrical shell subjected to compressive force acting concentrically:

$$F = -\sigma_c \frac{y^2}{2} + \frac{PR}{h}\frac{x^2}{2} + a_{11} \cos\frac{mx}{R} \cos\frac{ny}{R} + a_{22} \cos\frac{2mx}{R} \cos\frac{2ny}{R} + a_{02} \cos\frac{2ny}{R} + a_{20} \cos\frac{2mx}{R} + a_{13} \cos\frac{mx}{R} \cos\frac{3ny}{R} + a_{31} \cos\frac{3mx}{R} \cos\frac{ny}{R} \qquad (16)$$

The coefficients $a_{11}, a_{22}, a_{02}, a_{20}, a_{31}, a_{13}$ in Eqn. (16) were determined in terms of f_2, f_3, and f_4 from the compatibility equation as expressed in Eqns. 17(a-f):

$$\forall_{20} = \frac{a_{20}}{Eh^2} = (1 - Л)\frac{\beta}{4}\varphi_4 - (1 - Л^2)\left(\frac{\bar{\mu}^2}{32}{\varphi_2}^2\right) \quad (17a)$$

$$\forall_{02} = \frac{a_{02}}{Eh^2} = -(1 - Л^2)\frac{{\varphi_2}^2}{32\bar{\mu}^2} \qquad (17b)$$

$$\forall_{11} = \frac{a_{11}}{Eh^2} = (1 - Л)\frac{\varphi_2^2 \beta}{(1 + \bar{\mu}^2)^2} - (1 - Л^2)\frac{2\bar{\mu}^2}{(1 + \bar{\mu}^2)^2}\varphi_2(\varphi_3 + \varphi_4) \qquad (17c)$$

$$\forall_{22} = \frac{a_{22}}{Eh^2} = -(1 - Л^2)\frac{\bar{\mu}^2}{(1 + \bar{\mu}^2)^2}(\varphi_3\varphi_4) \qquad (17d)$$

$$\forall_{31} = \frac{a_{31}}{Eh^2} = -(1 - Л^2)\frac{2\bar{\mu}^2}{(9 + \bar{\mu}^2)^2}(\varphi_2\varphi_4) \qquad (17e)$$

$$\forall_{13} = \frac{a_{13}}{Eh^2} = -(1 - Л^2)\frac{2\bar{\mu}^2}{(1 + 9\bar{\mu}^2)^2}(\varphi_2\varphi_3) \qquad (17f)$$

Where

$$\bar{\mu} = \frac{n}{m} \quad \beta = \frac{R}{m^2 h} \quad \varphi_i = \frac{f_i}{h}, \text{and } i = 2, 3, 4$$

$\bar{\mu}$ *is called wavelength ratio in axial and circumferential direction*

3 Expression of Total Potential for internally pressurized thin Cylindrical Shell under Axial Compression

The total potential of the system, $\prod$ is the sum of the strain energies and it is expressed as follows,

$$\prod = U_e + U_c + U_P + U_{b,k} + U_{b,j} + U_P + U_{T,k} + U_{T,j} \quad (18)$$

3.1 Minimization of Total Potential for internally pressurized thin Cylindrical Shell under Axial Compression

The total potential energy of internally pressurized thin cylindrical subjected to axial compression must be minimum when the structure is in equilibrium.

The variation of potential with respect to each of the arbitrary parameters vanished for equilibrium, this gave rise to Eqn. (19):

$$\frac{\partial \overline{\prod}}{\partial \varphi_2} = 0, \quad \frac{\partial \overline{\prod}}{\partial \varphi_3} = 0, \quad \frac{\partial \overline{\prod}}{\partial \varphi_4} = 0 \qquad (19)$$

Evaluation of $\frac{\partial \overline{\prod}}{\partial \varphi_2} = 0$, $\frac{\partial \overline{\prod}}{\partial \varphi_3} = 0$, $\frac{\partial \overline{\prod}}{\partial \varphi_4} = 0$ yielded Eqns (20) – (22):

$$\phi_1 \frac{\beta}{1 - Л} = \Lambda_1 + \beta^2(\Lambda_2 + \Lambda_3\lambda + \Lambda_4\lambda^2) + \phi_2^2 \Lambda_5 \qquad (20)$$

$$\phi_2 \frac{\beta}{1 - Л} = B_1 + \beta^2 B_2 \lambda^2 + \phi_2^2\left(B_3 + \frac{B_4}{\lambda}\right) \qquad (21)$$

$$\phi_3 \frac{\beta}{1 - Л} = \aleph_1 + \beta^2(\aleph_2 + \aleph_3\lambda^2) + \phi_2^2\left(\aleph_4 + \frac{\aleph_5}{\lambda}\right) \qquad (22)$$

From Eqn. (20) the notations used were defined as follows;

$$\phi_1 = \bar{\sigma}_c - \bar{\mu}^2 \bar{P} \qquad (23a)$$

$$\Lambda_1 = \frac{(1 + \bar{\mu}^2)^2}{12(1 - \mu^2)} + \Psi_1 \qquad (23b)$$

$$\Lambda_2 = \frac{1}{(1+\bar{\mu}^2)^2} \qquad (23c)$$

$$\Lambda_3 = -\left[\frac{2(2+Л)(1+\lambda_1)}{(1+\bar{\mu}^2)^2} + \frac{\lambda_1}{2}\right]\bar{\mu}^2 \qquad (23d)$$

$$\Lambda_4 = 4(1+Л)\bar{\mu}^4\left[\frac{(1+\lambda_1)^2}{(1+\bar{\mu}^2)} + \frac{\lambda_1}{(9+\bar{\mu}^2)^2} + \frac{1}{(1+9\bar{\mu}^2)^2}\right] \qquad (23e)$$

$$\Lambda_5 = (1+Л)\left(\frac{1+\bar{\mu}^4}{16}\right) \qquad (23f)$$

$$\Psi_1 = \sum_{k=1}^{N_k}\frac{\bar{E}_k\bar{I}_k\bar{L}_k}{2}(C_1^4 + 6\bar{\mu}^2C_1^2S_1^2 + \bar{\mu}^4S_1^4) + \sum_{j=1}^{N_j}\frac{\bar{E}_j\bar{I}_j\bar{L}_j}{2}(C_2^4 + 6\bar{\mu}^2C_2^2S_2^2 + \bar{\mu}^4S_2^4 + \sum_{k=1}^{N_k}\frac{\bar{E}_k\bar{I}_k\bar{L}_k}{4}[(C_1+\bar{\mu}S_1)^2(S_1-\bar{\mu}C_1)^2 + (C_1-\bar{\mu}S_1)^2(S_1+\bar{\mu}C_1)^2] + \sum_{j=1}^{N_j}\frac{\bar{E}_j\bar{I}_j\bar{L}_j}{4}[(C_2+\bar{\mu}S_2)^2(S_2-\bar{\mu}C_2)^2 + (C_2-\bar{\mu}S_2)^2(S_2+\bar{\mu}C_2)^2] \qquad (23g)$$

The notations used in Eqn. (21) were defined as follows;

$$\phi_2 = -\bar{\mu}^2\bar{P} \qquad (24a)$$

$$B_1 = \frac{\bar{\mu}^4}{3(1-\mu^2)} + \Psi_2 \qquad (24b)$$

$$B_2 = (1+Л)\frac{2\bar{\mu}^4}{(1+\bar{\mu}^2)^2}\lambda_1^2 \qquad (24c)$$

$$B_3 = (1+Л)\left[\frac{1}{(1+9\bar{\mu}^2)^2} + \frac{(1+\lambda_1)}{(1+\bar{\mu}^2)^2}\right]\frac{\bar{\mu}^4}{2} \qquad (24d)$$

$$B_4 = -\frac{\bar{\mu}^2}{4(1+\bar{\mu}^2)^2} \qquad (24e)$$

$$\Psi_2 = \sum_{k=1}^{N_k}\bar{E}_k\bar{I}_k\bar{L}_k\,S_1^4\bar{\mu}^4 + \sum_{j=1}^{N_j}\bar{E}_j\bar{I}_j\bar{L}_j\,S_2^4\bar{\mu}^4 + \sum_{k=1}^{N_k}\bar{G}_k\bar{J}_k\bar{L}_k\,C_1^2S_1^2\bar{\mu}^4 + \sum_{j=1}^{N_j}\bar{G}_j\bar{J}_j\bar{L}_j\,C_2^2S_2^2\bar{\mu}^4 \qquad (3.46f)$$

From Eqn. (22), the notation used as obtained from Eqn. (3.41) were defined as follows;

$$\phi_3 = \bar{\sigma}_c \qquad (25a)$$

$$\aleph_1 = \frac{1}{3(1-\mu^2)} + \Psi_3 \qquad (25b)$$

$$\aleph_2 = \frac{1}{4} \qquad (25c)$$

$$\aleph_3 = (1+Л)\frac{2\bar{\mu}^4}{(1+\bar{\mu}^2)^2} \qquad (25d)$$

$$\aleph_4 = (1+Л)\frac{\bar{\mu}^4}{2}\left[\frac{1}{(9+\bar{\mu}^2)^2} + \left(1+\frac{1}{\lambda_1}\right)\frac{1}{(1+\bar{\mu}^2)^2}\right] \qquad (25e)$$

$$\aleph_5 = -\left[\frac{1}{(1+\bar{\mu}^2)^2} + \frac{1+Л}{8}\right]\frac{\bar{\mu}^2}{4\lambda_1} \qquad (25f)$$

$$\Psi_3 = \sum_{k=1}^{N_k}\bar{E}_k\bar{I}_k\bar{L}_k\,C_1^4 + \sum_{j=1}^{N_j}\bar{E}_j\bar{I}_j\bar{L}_j\,C_2^4 + \sum_{k=1}^{N_k}\bar{G}_k\bar{J}_k\bar{L}_k\,C_1^2S_1^2 + \sum_{j=1}^{N_j}\bar{E}_j\bar{I}_j\bar{L}_j\,C_2^2S_2^2 \qquad (25g)$$

Where

$\lambda = \frac{\phi_3}{\beta}$, $\lambda_1 = \frac{\phi_4}{\phi_3}$, $C_1 = \cos\gamma_1$, $C_2 = \cos\gamma_2$, $S_1 = \sin\gamma_1$, $S_2 = \sin\gamma_2$, $\bar{P} = \frac{PR^2}{Eh^2}$ and $\bar{\sigma}_c = \frac{\sigma_c R}{Eh}$

Eliminating φ_2 and β from Eqns. (3.42), (3.43) and (3.44), the following equation was obtained:

$$M_1\bar{\sigma}_c^2 + M_2\bar{\sigma}_c + M_3 = 0 \qquad (26)$$

where

$$M_1 = \frac{1}{(1-Л^2)}\left[\frac{\eta_2\eta_3}{\varpi_3^2}\left(\aleph_4 + \frac{\aleph_5}{\lambda} - \Lambda_5\right)^2 + \frac{\varpi_2}{\varpi_3}\left(B_3 + \frac{B_4}{\lambda}\right)^2 - \left(\frac{\varpi_2\eta_3}{\varpi_3^2} + \frac{\eta_2}{\varpi_3}\right)\left(\aleph_4 + \frac{\aleph_5}{\lambda} - \Lambda_5\right)\left(B_3 + \frac{B_4}{\lambda}\right)\right] \qquad (27a)$$

$$M_2 = -\frac{\bar{P}\bar{\mu}^2}{(1-Л^2)}\left\{\frac{2\eta_2\eta_3}{\varpi_3^2}\left(\aleph_4 + \frac{\aleph_5}{\lambda}\right)\left(\aleph_4 + \frac{\aleph_5}{\lambda} - \Lambda_5\right) + \frac{2\varpi_2}{\varpi_3}\left(B_3 + \frac{B_4}{\lambda} - \Lambda_5\right) - \left(\frac{\varpi_2\eta_3}{\varpi_3^2} + \frac{\eta_2}{\varpi_3}\right)\left[2\left(B_3 + \frac{B_4}{\lambda}\right)\left(\aleph_4 + \frac{\aleph_5}{\lambda}\right) + \Lambda_5^2 - \Lambda_5\left(B_3 + \frac{B_4}{\lambda} + \aleph_4 + \frac{\aleph_5}{\lambda}\right)\right]\right\} \qquad (27b)$$

$$M_3 = \frac{\bar{P}\bar{\mu}^4}{(1-Л^2)}\left[\frac{\eta_2\eta_3}{\varpi_3^2}\left(\aleph_4 + \frac{\aleph_5}{\lambda}\right)^2 + \frac{\varpi_2}{\varpi_3}\left(B_3 + \frac{B_4}{\lambda} - \Lambda_5\right)^2 - \left(\frac{\varpi_2\eta_3}{\varpi_3^2} + \frac{\eta_2}{\varpi_3}\right)\left(\aleph_4 + \frac{\aleph_5}{\lambda}\right) + \left(B_3 + \frac{B_4}{\lambda} - \Lambda_5\right)\right] + \frac{\eta_3^2\varpi_2^2}{\varpi_3^2} - \frac{2\eta_2\eta_3\varpi_2}{\varpi_3} + \eta_2^2 \qquad (27c)$$

Where η_2, and η_3 were defined as follows as derived:

$$\eta_2 = \Lambda_1 \left(B_3 + \frac{B_4}{\lambda}\right) - B_1\Lambda_5 \qquad (28a)$$

$$\eta_3 = (\Lambda_2 + \Lambda_3\lambda + \Lambda_4\lambda^2)\left(B_3 + \frac{B_4}{\lambda}\right) - B_2\Lambda_5\lambda^2 \qquad (28b)$$

Also, ϖ_2 and ϖ_3 were defined as follows:

$$\varpi_2 = \Lambda_1 \left(\aleph_4 + \frac{\aleph_5}{\lambda}\right) - \aleph_1\Lambda_5 \qquad (29a)$$

$$\varpi_3 = (\Lambda_2 + \Lambda_3\lambda + \Lambda_4\lambda^2)\left(\aleph_4 + \frac{\aleph_5}{\lambda}\right) - (\aleph_2 + \aleph_3\lambda^2)\Lambda_5 \qquad (29b)$$

Equation (26) is the governing equation for determining the critical buckling stress of an internally pressurized thin cylindrical shell loaded with axial compressive force and reinforced with inclined stiffeners.

4 RESULTS AND DISCUSSIONS

4.1 RESULTS

NUMERICAL EXAMPLES

The numerical analysis of this type of cylindrical shell was done by taking the following assumptions: $\bar{E}_k\bar{I}_k\bar{L}_k = \bar{E}_j\bar{I}_j\bar{L}_j$, $\bar{G}_k\bar{J}_k\bar{L}_k = \bar{G}_j\bar{J}_j\bar{L}_j$, $\Upsilon_1 = \Upsilon_2 = \Upsilon\,(for\,\Upsilon = 10^0, 20^0, 30^0, 40^0, 45^0, 50^0, 60^0)$, $\lambda = \lambda_1$, $m = 5$, $\bar{\mu} = 1$, $h = 0.05\,metre$, $\bar{P} = 2\,and\,R = 2\,metres$. Using the governing equation in Eqn (43) and the notation described from Eqn (27a) to Eqn (29b), the following data shown in Table 1 were obtained for different imperfect ratio, Л.

Table 1: Values of Buckling Stress Parameter, $\bar{\boldsymbol{\sigma}}_{\boldsymbol{c}}$ for different imperfect ratio for internally pressurized thin cylinders reinforced with inclined stiffeners subjected to uniform axial compression

IMPERFECT RATIO, Л	BUCKLING STRESS PARAMETER, $\bar{\sigma}_c$ OF STIFFENERS AT DIFFERENT ANGLES						
	10^0	20^0	30^0	40^0	45^0	50^0	60^0
0	5.6624	4.0054	1.2553	0.3941	0.3394	0.4505	1.1069
0.1	7.5788	4.6957	1.4670	0.4122	0.3575	0.5122	1.3826
0.2	8.0688	5.3574	1.7232	0.4434	0.3795	0.5708	1.6531
0.3	8.4725	5.9337	1.9976	0.4838	0.4032	0.6224	1.9030
0.4	8.6372	6.3810	2.2657	0.5310	0.4274	0.6629	2.1060
0.5	8.6698	6.6607	2.4493	0.5835	0.4517	0.6872	2.2265
0.6	8.4874	6.7322	2.6652	0.6395	0.4756	0.6894	2.2107
0.7	8.0403	6.5440	2.7236	0.6971	0.499	0.6626	2.0012
0.8	7.2397	6.0177	2.6265	0.7535	0.5217	0.5990	1.5520
0.9	5.9043	5.0061	2.3134	0.8055	0.5435	0.4902	0.8552

4.2 DISCUSSION OF RESULTS

The data in Table1 showed that as imperfect ratio of stiffeners inclined at 10^0 and 60^0 respectively increases from 0.1 to 0.5, its buckling stress parameter increases. For stiffeners inclined at 20^0 and 50^0 respectively, there was progressive increase of their buckling stress parameter from imperfect ratio of 0.1 to imperfect ratio of 0.6. While, stiffeners inclined at 30^0 have progressive increase of their buckling stress parameter from imperfect ratio of 0.1 to imperfect ratio of 0.7.

However, stiffeners inclined at 40^0 and 45^0 respectively have progressive increase of their buckling stress parameter from imperfect ratio of 0.1 to imperfect ratio of 0.9. Buckling stress parameter is least at 45^0 inclined stiffeners and maximum at 10^0 inclined stiffeners for all imperfect ratios considered. The results in Table 1 also showed that 10^0 inclined stiffeners have maximum critical buckling stress at imperfect ratio of 0.5, while 45^0 inclined stiffeners have the least critical buckling stress at imperfect ratio of 0.1.

5 CONCLUSION

With reference to the results obtained in this research, engineers designing cylindrical shell structures with the aim of providing resistance to buckling would be able to select suitable inclined stiffeners for the structure under uniform axial compression.

REFERENCES

[1] Calladine, C. R (2007): Theory of Shell Structures, Cambridge University Press, New York

[2] Ventsel, E and Krauthammer (2001): Thin Plates and Shells-Theory, Analysis and Applications. Marcel Dekker Inc. New York.

[3] Iyengar (1998): Structural Stability of Columns and Plates, John Wiley and Sons, New York.

[4] Houliara, S. (2008): Computational Techniques in Structural Stability of Thin-walled Cylindrical shells, Ph.D Thesis in Mechanical and Industrial Engineering, University of Thessaly.

[5] Sosa, E. N. (2005): Computational Buckling Analysis Cylindrical Thin-walled Above Ground Tanks, Ph.D Thesis in Civil Engineering, University of Puerto Rico.

[6] Catellani, G., Pellicano, F., Dall' Asla, D., and Amabili, M. (2004): Parametric Instability of a Circular Cylindrical Shell with Geometric Imperfections, Comput. Struct, Vol. 82, pp 31-32.

[7] Arani, G., Golabi, S., Loghman, A., and Danesh, H (2007): Investigating Elastic Stability of Cylindrical Shell with Elastic Core under Axial Compression by Energy Method. Journal of Mechanical Science and Technology, vol. 21(7), 983-996

[8] Szilard, R. (2004). Theories and Application of Plate Analysis: Classical, Numerical and Engineering Method. John Wiley and Sons inc., New Jersey.

[9] Von, K and Tsien, E. (1941): The Buckling of Thin Cylindrical Shells under Axial Compression. Journal of Aero Science, vol.8

[10] Chai, H. Y. and Sung, C. L. (2011) Stability of Structures, Principles and Applications, Butterworth-Heinemann, Oxford, U.K.

[11] George, J. S. (1996). Buckling and Postbuckling of Imperfect Cylindrical Shells: A Review, Applied Mechanics Reviews, vol. 39 No.10

Notations	**Meaning**
$\in_x, \in_y$	Strains in x and y
w_0	Initial deflection
w	Total radial deflection
E	Young's modulus of elasticity of the shell
D	Flexural rigidity of the shell
h	Thickness of the shell
μ	Poisson ratio
Л	Imperfect ratio
R	Radius of the cylindrical shell
F	Airy's stress function
E_j	Young's modulus of elasticity of j^{th} stiffeners
E_k	Young's modulus of elasticity of k^{th} stiffeners
G_k	Shear modulus of k^{th} stiffeners
G_j	Shear modulus of j^{th} stiffeners
I_j	moment of inertia of j^{th} stiffeners
I_k	moment of inertia of k^{th} stiffeners
J_k	polar moment of inertia of k^{th} stiffeners
J_j	polar moment of inertia of j^{th} stiffeners
L	Length of the cylindrical shell
L_k	Length of k^{th} stiffeners
L_j	Length of j^{th} stiffeners
$\bar{L}_J$	Dimensionless length of j^{th} stiffeners
$\bar{L}_k$	Dimensionless length of k^{th} stiffeners
$\bar{E}_k$	Dimensionless Young's modulus of elasticity of k^{th} stiffeners
$\bar{E}_j$	Dimensionless Young's modulus of elasticity of j^{th} stiffeners
$\bar{G}_j$	Dimensionless shear modulus of j^{th} stiffeners
$\bar{G}_k$	Dimensionless shear modulus of k^{th} stiffeners
$\bar{I}_j$	Dimensionless moment of inertia of j^{th} stiffeners

$\bar{I}_k$	Dimensionless moment of inertia of k[th] stiffeners
$\bar{J}_j$	Dimensionless polar moment of inertia of j[th] stiffeners
$\bar{J}_k$	Dimensionless polar moment of inertia of k[th] stiffeners
m	Number of waves in axial direction
n	Number of waves in circumferential direction
γ_1	Inclination angle of the K[th] stiffeners parallel with y_1-line and normal to y_2^1-line
γ_2	Inclination angle of the j[th] stiffeners parallel with y_2-line and normal to y_1^1-line
$U_{T,j}$	Torsional strain energy for j[th] stiffeners inclined at angle, γ_2
$U_{T,k}$	Torsional strain energy for K[th] stiffeners inclined at angle, γ_1
$U_{b,j}$	Bending strain energy for j[th] stiffeners inclined at angle, γ_2
$U_{b,k}$	Bending strain energy for K[th] stiffeners inclined at angle, γ_1
U_c	Strain energy due to external force applied at the ends of the shell
U_e	Extensional strain energy in the shell
U_P	Strain energy due to internal pressure
U_m	Potential due to edge bending due to application of eccentric loading
σ_b	Bending stress
∇^4	Biharmonic operator
∇^2	Laplace operator
P	Internal pressure
$\bar{\mu}$	Wavelength ratio
$\bar{P}$	Dimensionless internal pressure
u, v, w	Components of displacements in x, y, z directions
x, y, z	Orthogonal coordinates on median surface of the shell
β	Dimensionless parameter that connect h, R and m
u, v, w	Components of displacements in x, y, z directions
x, y, z	Orthogonal coordinates on median surface of the shell
β	Dimensionless parameter that connect h, R and m
C_1	Cosine of angle γ_1

ISSN 2229-5518

S_1	Sine of angle γ_1
C_2	cosine of angle γ_2
S_2	Sine of angle γ_2
λ , λ_1	Deflection parameters
N_K	Number of stiffeners in γ_1 -direction
N_J	Number of stiffeners in γ_2 –direction
$\prod$	Total strain energies
$\bar{\prod}$	Non-dimensional total strain energies
$\bar{U}$	Non-dimensional strain energy